ADIKAVI NANNAYA UNIVERSITY

B.A./B.Sc. THIRD YEAR MATHEMATICS SYLLABUS

SEMESTER – V, PAPER -5

RING THEORY & VECTOR CALCULUS

60 Hrs

UNIT – 1 (12 hrs) RINGS-I : -

Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws Rings, Integral Domains, Division Ring and Fields, The characteristic of a ring - The characteristic of an Integral Domain, The characteristic of a Field. Sub Rings, Ideals

UNIT – 2 (12 hrs) RINGS-II : -

Definition of Homomorphism – Homorphic Image – Elementary Properties of Homomorphism – Kernel of a Homomorphism – Fundamental theorem of Homomorhphism –
Maximal Ideals – Prime Ideals.

UNIT –3 (12 hrs) VECTOR DIFFERENTIATION : -

Vector Differentiation, Ordinary derivatives of vectors, Differentiability, Gradient, Divergence, Curl operators, Formulae Involving these operators.

UNIT – 4 (12 hrs) VECTOR INTEGRATION : -

Line Integral, Surface Integral, Volume integral with examples.

UNIT – 5 (12 hrs) VECTOR INTEGRATION APPLICATIONS : -

Theorems of Gauss and Stokes, Green's theorem in plane and applications of these theorems.

Reference Books :-

1. Abstract Algebra by J. Fralieh, Published by Narosa Publishing house.
2. Vector Calculus by Santhi Narayana, Published by S. Chand & Company Pvt. Ltd., New Delhi.
3. A text Book of B.Sc., Mathematics by B.V.S.S.Sarma and others, published by S. Chand & Company Pvt. Ltd., New Delhi.
4. Vector Calculus by R. Gupta, Published by Laxmi Publications.
5. Vector Calculus by P.C. Matthews, Published by Springer Verlag publicattions.
6. Rings and Linear Algebra by Pundir & Pundir, Published by Pragathi Prakashan.

Suggested Activities:

Seminar/ Quiz/ Assignments/ Project on Ring theory and its applications

ADIKAVI NANNAYA UNIVERSITY

B.A./B.Sc. THIRD YEAR MATHEMATICS SYLLABUS

SEMESTER – V, PAPER -6

LINEAR ALGEBRA

60 Hrs

UNIT – I (12 hrs) : Vector Spaces-I :

Vector Spaces, General properties of vector spaces, n-dimensional Vectors, addition and scalar multiplication of Vectors, internal and external composition, Null space, Vector subspaces, Algebra of subspaces, Linear Sum of two subspaces, linear combination of Vectors, Linear span Linear independence and Linear dependence of Vectors.

UNIT –II (12 hrs) : Vector Spaces-II :

Basis of Vector space, Finite dimensional Vector spaces, basis extension, co-ordinates, Dimension of a Vector space, Dimension of a subspace, Quotient space and Dimension of Quotientspace.

UNIT –III (12 hrs) : Linear Transformations :

Linear transformations, linear operators, Properties of L.T, sum and product of LTs, Algebra of Linear Operators, Range and null space of linear transformation, Rank and Nullity of linear transformations – Rank – Nullity Theorem.

UNIT –IV (12 hrs) : Matrix :

Linear Equations, Characteristic Roots, Characteristic Values & Vectors of square Matrix, Cayley – Hamilton Theorem.

UNIT –V (12 hrs) : Inner product space :

Inner product spaces, Euclidean and unitary spaces, Norm or length of a Vector, Schwartz inequality, Triangle in Inequality, Parallelogram law, Orthogonality, Orthonormal set, complete orthonormal set, Gram – Schmidt orthogonalisation process. Bessel's inequality and Parseval's Identity.

Reference Books :

1. Linear Algebra by J.N. Sharma and A.R. Vasista, published by Krishna Prakashan Mandir, Meerut-250002.
2. Matrices by Shanti Narayana, published by S.Chand Publications.
3. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson Education (low priced edition), New Delhi.
4. Linear Algebra by Stephen H. Friedberg et al published by Prentice Hall of India Pvt. Ltd. 4th Edition 2007.

Suggested Activities:

Seminar/ Quiz/ Assignments/ Project on "Applications of Linear algebra Through Computer Sciences"

MATHEMATICS MODEL PAPER

FIFTH SEMESTER

PAPER 5 – RING THEORY & VECTOR CALCULUS

COMMON FOR B.A & B.Sc

(w.e.f. 2015-16 admitted batch)

Time: 3 Hours **Maximum Marks: 75**

SECTION-A

Answer any **FIVE** questions. Each question carries **FIVE** marks. **5 x 5 = 25 Marks**

1) Prove that every field is an integral domain.

2) If R is a Boolean ring then prove that (i) a + a = 0 $\forall$ $a \in R$ (ii) a + b = 0 $\Rightarrow$ a = b.

3) Prove that Intersection of two sub rings of a ring R is also a sub ring of R.

4) If $\boldsymbol{f}$ is a homomorphism of a ring R into a ring R^1 then prove that Ker $\boldsymbol{f}$ is an ideal of R.

5) Prove that Curl (grad $\emptyset$) = $\bar{0}$.

6) If $\boldsymbol{f} = xy^2\, \boldsymbol{i} + 2x^2\, yz\, \boldsymbol{j} - 3yz^2\, \boldsymbol{k}$ the find div $\boldsymbol{f}$ and Curl $\boldsymbol{f}$ at the point (1, -1 1).

7) If $R(u) = (u - u^2)i + 2u^3 j - 3k$ then find $\int_1^2 R(u)\,du$.

8) Show that $\int_s (ax\,i + by\,j + cz\,k).N\,dS = 4\frac{\pi}{3}(a + b + c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

SECTION-B

Answer the all **FIVE** questions. Each carries TEN marks. **5 x 10 = 50 Marks**

9(a) Prove that a finite integral domain is a field

OR

9(b) Prove that the characteristic of an integral domain is either a prime or zero.

10(a) State and prove fundamental theorem of homomorphism of rings.

OR

10(b) Prove that the ring of integers Z is a Principal ideal ring.

11(a) If $a = x + y + z$, $b = x^2 + y^2 + z^2$, $c = xy + yz + zx$; then prove that [grad a, grad b, grad c] = 0.

OR

11(b) Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$, $z = t^3$ at the point (1, 1, 1,).

12(a) Evaluate $\int_S F.N ds$, where $F = z\,\mathbf{i} + x\,\mathbf{j} - 3y^2z\,\mathbf{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$, and $z = 5$.

OR

12(b) If $F = \left(2x^2 - 3z\right)i - 2xy\,j - 4x\,k$,then evaluate $\iiint_V \nabla . F\,dV$ where V is the closed region bounded by the planes $x = 0, y = 0, z = 0 \; and \; 2x + 2y + z = 4$.

13(a) State and Prove Stoke's theorem.

OR

13(b) Find $\oint_C (x^2 - 2xy)\, dx + (x^2y + z)\, dy$ around the boundary C of the region bounded by $y^2 = 8x$ and $x = 2$ by Green's theorem.

MATHEMATICS MODEL PAPER
FIFTH SEMESTER
PAPER 6 – LINEAR ALGEBRA
COMMON FOR B.A & B.Sc
(w.e.f. 2015-16 admitted batch)

Time: 3 Hours **Maximum Marks: 75**

SECTION-A

Answer any **FIVE** questions. Each question carries **FIVE** marks. **5 x 5 = 25 Marks**

1) Let p, q, r be the fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F, such that $px + qy + rz = 0$ is a vector subspace of V_3 (F).
2) Express the vector $\alpha = (1, -2, 5)$ as a linear combination of the vectors $e_1 = (1, 1, 1)$, $e_2 = (1, 2, 3)$ and $e_3 = (2, -1, 1)$.
3) If α, β, γ are L.I vectors of the vector space V(R) then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ are also L.I vectors.
4) Describe explicitly the linear transformation T: $R^2 \rightarrow R^2$ such that T (1, 2) = (3, 0), and T (2, 1) = (1,2).
5) Let U(F) and V(F) be two vector spaces and $T: U(F) \rightarrow V(F)$ be a linear transformation. Prove that the range set R(T) is a subspace of V(F).
6) Solve the system 2x-3y+z=0, x+2y-3z=0, 4x-y-2z=0.
7) State and prove Schwarz inequality.
8) Show that the set $S = \{(1,1,0), (1,-1,1), (-1,1,2)\}$ is an orthogonal set of the inner product space R^3(R).

SECTION-B

Answer the all **FIVE** questions. Each carries TEN marks. **5 x 10 = 50 Marks**

9(a) Prove that the subspace W to be a subspace of V(F) $\Leftrightarrow a\alpha + b\beta \in W\ \forall a, b \in F$ and $\alpha, \beta \in W$.

OR

9(b) Prove that the four vectors α =(1,0,0), β =(0,1,0), γ =(0,0,1), δ =(1,1,1) in V3(C) form a Linear dependent set, but any three of them are Linear Independent.

10(a) Let W be a subspace of a finite dimensional vector space V(F), then prove that

$$\mathbf{dim}\left(V/W\right) = \mathbf{dim}(V) - \mathbf{dim}(W)$$

OR

10(b) Let W_1 and W_2 be two subspaces of a finite dimensional vector space V(F). Then prove that

$$\mathbf{dim}(W_1 + W_2) = \mathbf{dim}(W_1) + \mathbf{dim}(W_2) - \mathbf{dim}(W_1 \cap W_2).$$

11(a) State and prove Rank-Nullity theorem.

OR

11(b) Define a Linear transformation. Show that the mapping T: $R^3 \rightarrow R^2$ is defined by T (x, y, z) = (x - y, x - z) is a linear transformation.

12(a) State and prove Cayley- Hamilton theorem.

OR

12(b) Find the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

13(a) State and prove Bessel's inequality.

OR

13(b) Applying Gram-Schmidt orthogonalization process to obtain an orthonormal basis of $R^3(R)$ from the basis S = { (1, 1, 0), (-1,1,0), (1, 2, 1,)}.
